Hong Kong Mathematics Olympiad (2017/18)
Heat Event (Group)
香港數學競賽 (2017/18)
初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 設 $f(x)$ 為二次多項式，其中 $f(1)=\frac{1}{2}$，$f(2)=\frac{1}{6}$，$f(3)=\frac{1}{12}$。求 $f(6)$ 的值。

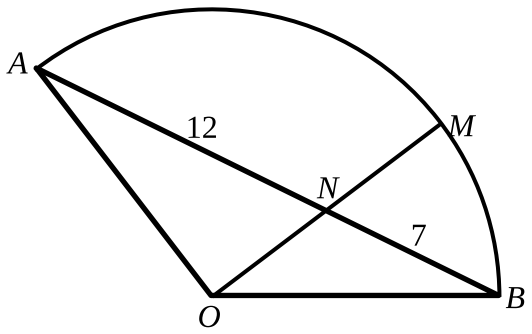
   Let $f(x)$ be a polynomial of degree 2, where $f(1)=\frac{1}{2}$, $f(2)=\frac{1}{6}$, $f(3)=\frac{1}{12}$. Find the value of $f(6)$.

2. 求 $\sqrt{2018\times 2012\times 1988\times 1982+8100}$。

   Evaluate $\sqrt{2018\times 2012\times 1988\times 1982+8100}$.

3. 如圖一所示，OAB 是一個以 O 為圓心的圓的扇形。N 則為半徑 OM 與 AB 的交點。已知 $AN=12$，$BN=7$ 及 $3ON=2MN$。求 OM 的長度。

   As shown in Figure 1, OAB is a sector of a circle with centre O. N is the intersecting point of radius OM and AB. Given that $AN=12$, $BN=7$ and $3ON=2MN$. Find the length of OM.



圖一

Figure 1

4. 對任意非零實數 x，函數 $f(x)$ 有以下特性：$2f(x)+f(\frac{1}{x})=11x+4$。設 S 為所有滿足於 $f(x)=2018$ 的根之和。求 S 的值。

   For any non-zero real number x, the function $f(x)$ has the following property: $2f(x)+f(\frac{1}{x})=11x+4$. Let S be the sum of all roots satisfying the equation $f(x)=2018$. Find the value of S.

5. 求可滿足下列方程組的 x 值：

$$\begin{cases} x^2+9x-10y-220=0 \\ y^2-5x+6y-166=0 \\ xy=195 \end{cases}$$

Find the value of x that satisfy the following system of equations:

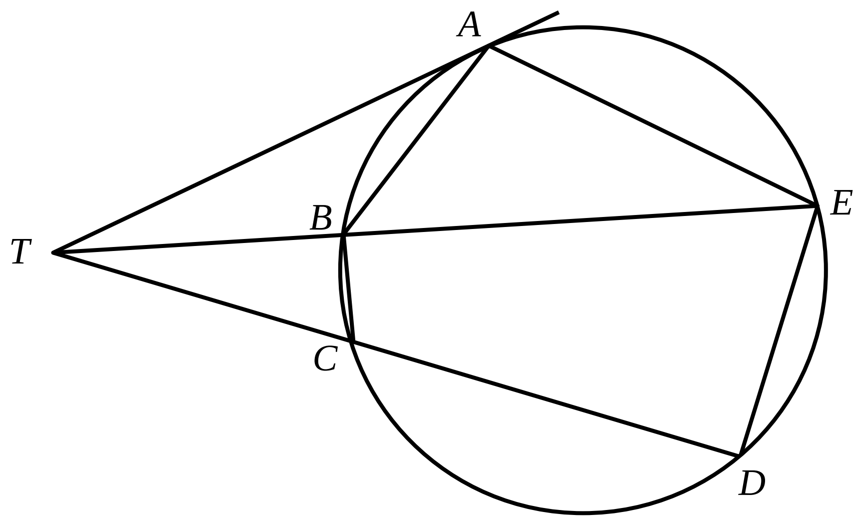
$$\begin{cases} x^2+9x-10y-220=0 \\ y^2-5x+6y-166=0 \\ xy=195 \end{cases}$$

6. 已知 $n^4+104=3^m$，其中 n、m 為正整數。求 n 的最小值。

Given that $n^4+104=3^m$, where n , m are positive integers. Find the least value of n .

7. 如圖二所示，A、B、C、D 及 E 為圓上的點。T 是該圓外的一點。TA 是該圓在點 A 的切線，TBE 及 TCD 為直線。已知 TBE 是$\angle ATD$ 的角平分線、$TA=12$、 $TB=6$ 及 $TC=8$。求 ΔABE 與四邊形 $BCDE$ 的面積比。

As shown in Figure 2, A, B, C, D and E are points on the circle. T is a point outside the circle such that TA is tangent to the circle at A and TBE and TCD are straight lines. It is given that TBE is the angle bisector of $\angle ATD$, TA = 12, TB = 6 and TC = 8. Find the ratio of the area of ΔABE to the area of quadrilateral $BCDE$.



圖二

Figure 2

8. 已知 a、b、c、d、e、f、g 及 h 為正整數，使得 $a>b>c>d>e>f>g>h$ 及 $a+h=b+g=c+f=d+e=35$，問有多少組可行答案 $\{a,b,c,d,e,f,g,h\}$ 存在？

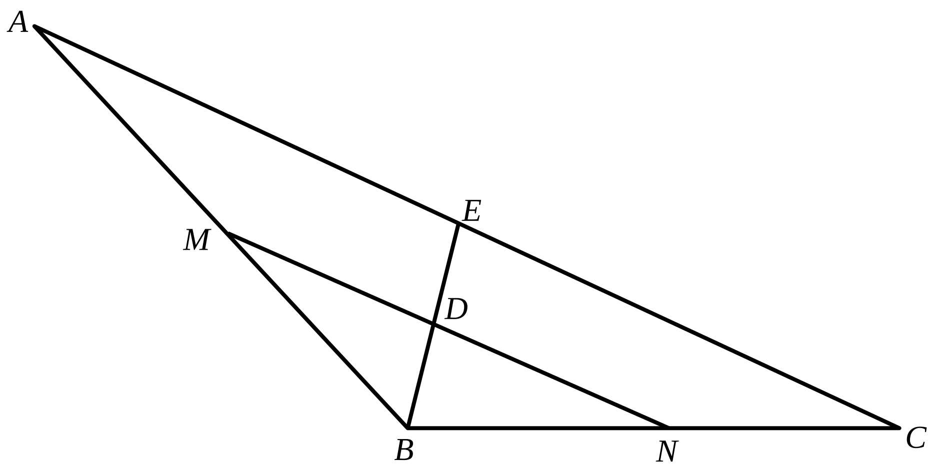
Given that a, b, c, d, e, f, g and h are positive integers such that $a>b>c>d>e>f>g>h$ and $a+h=b+g=c+f=d+e=35$. How many possible solution set of $\{a,b,c,d,e,f,g,h\}$ exist?

9. 求 $\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{100}\right)+\left(\frac{2}{3}+\frac{2}{4}+\cdots+\frac{2}{100}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{100}\right)+\cdots+\left(\frac{98}{99}+\frac{98}{100}\right)+\frac{99}{100}$ 的值。

Find the value of $\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{100}\right)+\left(\frac{2}{3}+\frac{2}{4}+\cdots+\frac{2}{100}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{100}\right)+\cdots+\left(\frac{98}{99}+\frac{98}{100}\right)+\frac{99}{100}$.

10. 如圖三所示，ABC 是一個三角形，其中 $AB=40$、$BC=30$ 及 $\angle ABC=150°$。M 及 N 分別為 AB 及 BC 的中點。$\angle ABC$ 的角度平分綫分別相交 MN 及 AC 於 D 及 E。求四邊形 $AMDE$ 的面積。

As shown in Figure 3, ABC is a triangle with $AB=40$, $BC=30$ and $\angle ABC=150°$. M and N are the mid-points of AB and BC respectively. The angle bisector of $\angle ABC$ intersects MN and AC at D and E respectively. Find the area of quadrilateral $AMDE$.



圖三

Figure 3

完

END